Persistent currents in a Moebius ladder: A test of interchain coherence of interacting electrons

Frédéric Mila[a], Charles Stafford[b] and Sylvain Capponi[a]

(a) *Laboratoire de Physique Quantique, Université Paul Sabatier, 31062 Toulouse (France)*

(b) *Institut de Physique Théorique, Université de Fribourg, 1700 Fribourg (Switzerland)*

Persistent currents in a Moebius ladder are shown to be very sensitive to the effects of intrachain interactions on the hopping of electrons between chains. Their periodicity as a function of flux is doubled for strong enough repulsive interactions because electrons cannot hop coherently between the chains and have to travel along the full edge of the Moebius ladder, thus encircling the flux twice. Mesoscopic devices that should enable one to observe these effects are proposed.

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The problem of transport in very anisotropic systems has become one of the central issues in the field of strongly-correlated systems. Roughly speaking, it can be stated as follows: Consider an electronic system with transfer integrals much smaller in one direction than in the other(s). Is it possible, and under which conditions, that transport is coherent in the highly conducting direction(s) and incoherent in the other direction at temperatures much smaller than the smallest transfer integrals? This question has recently been raised in several contexts, the most prominent examples being the high-T_c superconductors for quasi-2D systems [1,2], and the organic superconductors, the Bechgaard salts, for quasi-1D systems [3,4]. This property is clearly inconsistent with Fermi Liquid theory, and the best candidates to describe such a behaviour are models of strongly-correlated electrons. Metallic behaviour is characterized by the presence of a Drude peak in the optical conductivity at zero temperature, and what one is looking for is a model for which there is no Drude peak in the conductivity in one direction, while there is one in the other(s). The optical conductivity is not a simple object to calculate though, and a number of authors concentrate on the Green's functions and look for models with no pole in the Green's function in one direction and one in the other(s). Both properties are very likely related, although this is not totally clear due to vertex corrections.

So far, most results have been obtained for quasi-1D systems, i.e. systems of coupled Luttinger liquids. Renormalization Group (RG) arguments [5] suggest that there will be a pole in the transverse Green's function as long as the Luttinger liquid exponent α is smaller than 1, while another approach drawing an analogy to the problem of coherence in a two-level system coupled to a bath suggests that the pole in the transverse Green's functions might disappear much earlier [6]. Evidence in favor of the latter picture based on the analysis of the angular dependence of the magnetoresistance of the Bechgaard salt $(TMTSF)_2PF_6$ has been presented [3,4]. However, the issue remains controversial, and more direct evidence of the effect of correlations on the coherence of motion perpendicular to the chains would be highly desirable. In the case of two chains, coherence shows up as a splitting of the bonding and antibonding bands, which could in principle be measured in angular resolved photoemission experiments. However, considering the difficulty in interpreting photoemission experiments on low dimensional systems, and the need to vary parameters like α or the perpendicular hopping, this prospect remains remote to say the least.

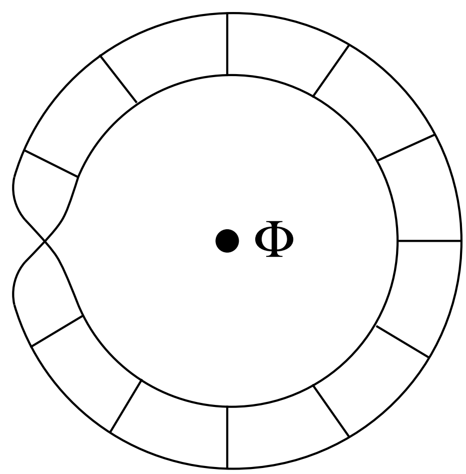

FIG. 1. Schematic diagram of a Moebius ladder pierced by a magnetic flux $(\hbar c/e)\Phi$.

In this paper, we propose a new approach to study the problem of interchain coherence based on the analysis of persistent currents in a Moebius ladder (see Fig. 1). If electrons can hop coherently from one edge to the other, the groundstate energy and the persistent current will be periodic functions of the flux with the usual period $\phi_0 = hc/e$. However, if the perpendicular hopping integral $t_\perp$ is switched off, electrons will have to go twice around the flux to reach the same site, and the period

will be $\phi_0/2$. Now, for an interacting system with $t_\perp \neq 0$, the period is expected to be ϕ_0 as long as coherent motion between the chains is possible, and to become $\phi_0/2$ when interactions are strong enough to prevent any coherent motion between the chains. Thus, measuring the periodicity of the persistent current should provide direct information on the coherence of interchain hopping.

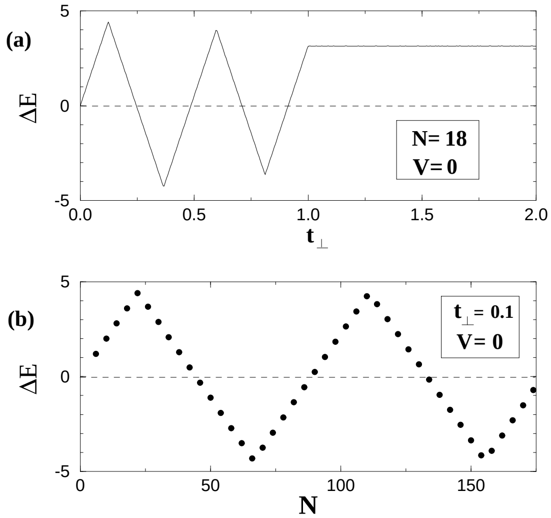

FIG. 2. ΔE for a non-interacting, quarter filled system: a) as a function of $t_\perp$; b) as a function of the number of rungs N.

This picture turns out to be essentially true, although the analysis is not as straightforward as one might hope due to rather interesting finite-size effects. Let us start with a careful analysis of the non-interacting case. For simplicity, we consider spinless fermions throughout this paper. The spin is not expected to play any significant role for this property, a point of view confirmed by preliminary results we have obtained for fermions with spin. Spinless fermions on a Moebius ladder with N rungs pierced by a magnetic flux $(\hbar c/e)\Phi$ can be described by a one-dimensional periodic Hamiltonian

$$H = -t \sum_{i=1}^{2N} (e^{i\Phi/N} c_i^\dagger c_{i+1} + h.c.) - \frac{1}{2} t_\perp \sum_{i=1}^{2N} (c_i^\dagger c_{i+N} + h.c.) \quad (1)$$

with the usual convention $c_{i+2N} = c_i$. This Hamiltonian is readily diagonalized by a Fourier transform, and the dispersion reads

$$\epsilon_k = -2t \cos(ka + \Phi/N) - t_\perp \cos(Nka) \quad (2)$$

with $k = p(2\pi/2Na)$, p integer. The essential ladder structure of the Moebius ladder is contained in this expression because $\cos(Nka) = \cos(p\pi) = +1$ if p is even and -1 if p is odd. The system thus consists of bonding and antibonding bands with the usual dispersions $\epsilon_k = -2t\cos(ka + \Phi/N) \pm t_\perp$. The difference with a standard ladder is that the wavevectors $k = p(2\pi/2Na)$ are restricted to even and odd values of p for the bonding and antibonding bands respectively.

Let us consider the periodicity of the groundstate energy of such a system as a function of Φ. If $t_\perp$ is large enough, all the fermions are in the bonding band, and the total energy reads $E(\Phi) = \sum_p [-2t\cos(p\pi/N + \Phi/N) - t_\perp]$, where the sum over p is restricted to *even* integers chosen to give the lowest energy for a given Φ. This function is clearly periodic in Φ with period 2π. If $t_\perp = 0$, the bonding and antibonding bands form a single band, and the total energy reads $E(\Phi) = -2t \sum_p \cos(p\pi/N + \Phi/N)$, where the integers p can now be both even and odd, and the periodicity of this function is π. For intermediate values of $t_\perp$, the periodicity is 2π except for specific values of $t_\perp$ where it is π. The number of such points scales with the number of particles in the system. To understand this, let us consider the quantity

$$\Delta E = (-1)^{\frac{n-1}{2}} N [E(\Phi = \pi) - E(\Phi = 0)] \quad (3)$$

where n is the number of particles. In the following, n is restricted to odd values to have a non degenerate groundstate which makes the analysis slightly simpler, although the results are essentially equivalent for an even number of particles, and the factor $(-1)^{\frac{n-1}{2}}$ has beeen included to insure that ΔE is always positive in the limit $t_\perp \to 0$. Besides, as for the curvature at $\Phi = 0$, which gives the Drude weight, one has to multiply by N for 1D systems to get non-vanishing results in the thermodynamic limit. A typical example of the behaviour of ΔE with $t_\perp$ is depicted in Fig. 2a. It is clear from the dispersion of Eq. (2) that both $E(\Phi = \pi)$ and $E(\Phi = 0)$, and hence ΔE, are piece-wise linear functions of $t_\perp$. The slope of ΔE changes each time a pair of particles goes from the antibonding band to the bonding band, which we know has to occur because the antibonding band is certainly empty for large enough $t_\perp$. Now these transitions do not occur for the same values of $t_\perp$ for $\Phi = \pi$ and $\Phi = 0$. As a result, the slope of ΔE alternates between $2N$ and $-2N$ until the antibonding band is empty, in which case it is of course 0. Between changes of slopes, ΔE vanishes once, which corresponds to points where the periodicity is π, like for $t_\perp = 0$. Since the particles change bands in pairs, the number of such points is essentially half the number of particles (more precisely $2n + 1$ including $t_\perp = 0$ for $4n + 1$ particles, and similar formulae for other fillings). The same effect shows up in the behaviour of ΔE with N: For intermediate values of $t_\perp$, ΔE oscillates as a function of size with extrema each time the difference in the particle number between the bonding and the antibonding bands increases by 2 (see Fig. 2b). The points lie on a piece-wise linear curve with a slope alternating between $2t_\perp$ and $-2t_\perp$. As a consequence, the periodicity of the groundstate energy for a given $t_\perp$ and for non

interacting fermions is not a well defined quantity in the thermodynamic limit.

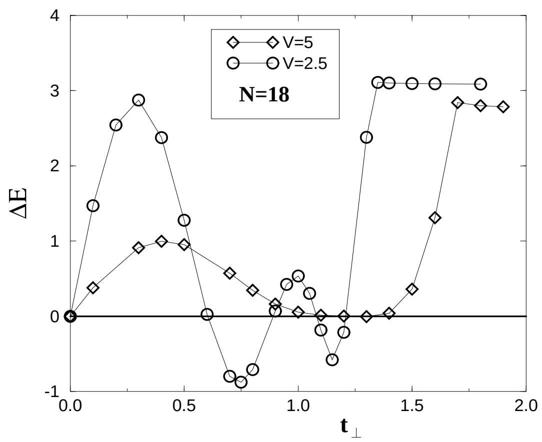

FIG. 3. ΔE as a function of $t_\perp$ for $V = 2.5$ ($\alpha = 0.36$) and $V = 5$ ($\alpha = 1$) for a quarter-filled system with 18 rungs. All energies are in units of t.

Let us now consider the effect of intrachain interactions on the periodicity of the groundstate energy. To describe systems with large values of the Luttinger liquid exponent α, we consider an interaction term of the form

$$H_{\rm int} = \sum_i \left(V n_i n_{i+1} + \frac{2V}{3} n_i n_{i+2} + \frac{V}{2} n_i n_{i+3} \right) \quad (4)$$

For a quarter-filled system, the exponent α of this model has already been calculated with standard techniques [7], and it reaches the value 1 for $V/t = 5$. Using Lanczos technique, we have calculated the dependence of ΔE on $t_\perp$ for different values of V/t and different sizes up to 36 sites ($N = 18$). Typical results are shown in Fig. 3. The effects of intrachain repulsion are rather dramatic. The first oscillation quickly becomes the dominant one, and the first value of $t_\perp$ where ΔE vanishes increases significantly: The curve is already strongly affected for $V/t = 2.5$ (Fig. 3) with respect to the non-interacting case (Fig. 2a). But more importantly, the oscillations disappear altogether for $V/t = 5$, i.e. $\alpha = 1$. This critical value turns out to be independent of the size. The origin of the oscillations being that particles go from the antibonding band to the bonding band, this means that these concepts have lost their meaning when α is big enough. In other words, $t_\perp$ is no longer able to produce two separate bands in the low energy spectrum of the system.

Our discussion of the dependence of ΔE on N for interacting systems is limited by the maximum size we can handle with Lanczos, namely 36 sites ($N = 18$) at quarter-filling. For $t_\perp = 0.1$, we are limited to the first linear section of Fig. 2b where the slope is equal to $2t_\perp$ in the non-interacting case. The slope is considerably reduced by interactions and *changes sign* between $V/t = 5$ and $V/t = 6$, which means that ΔE *decreases* with N (see Fig. 4a). Given the absence of oscillations in ΔE as a function of $t_\perp$ for these values of the interaction, it is natural to assume that ΔE does not change sign as a function of N either. This leads us to the conclusion that ΔE goes to 0 in the thermodynamic limit when $\alpha \geq 1$ [8]. Significant effects are already present for $\alpha < 1$, however. Let us consider for instance $t_\perp = 0.4$, for which ΔE changes sign between $N = 10$ and $N = 14$ in the non-interacting case (see Fig. 4b). For the sizes we can reach, ΔE already does not change sign for $V/t = 2$, and the curve is very flat for V/t as small as 3 ($\alpha = 0.46$). It is in fact possible that even in that case ΔE goes to 0 with damped oscillations. Obtaining numerical results on larger systems by other methods, e.g. Densitiy Matrix Renormalization Group, would be very useful to check this point. Let us note that oscillations would also be present for a regular ladder. However the periodicity remains equal to ϕ_0 even if $t_\perp$ is switched off. So the Moebius geometry is essential to observe the effects of interactions.

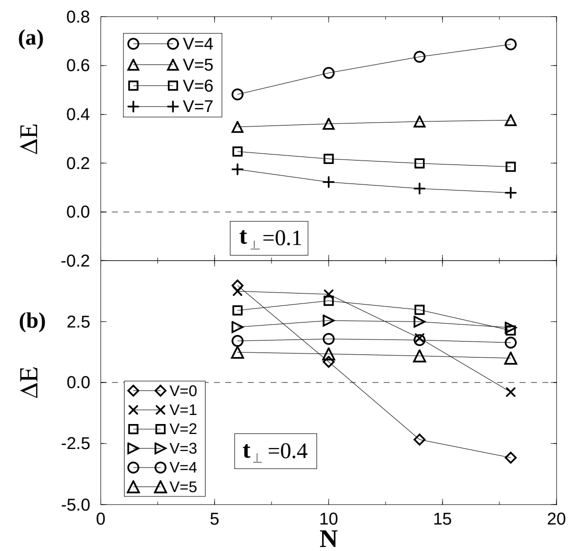

FIG. 4. ΔE as a function of the number of rungs N for different values of V and two values of $t_\perp$: a) $t_\perp = 0.1$; b) $t_\perp = 0.4$. All energies are in units of t.

To make contact with current theories of the effect of interactions on interchain hopping, let us first note that the present results strongly suggest that the system has lost any memory of the splitting between bonding and antibonding bands when $\alpha = 1$. This is reminiscent of the RG result that $t_\perp$ is an irrelevant perturbation when $\alpha > 1$, but it is much stronger: RG arguments are limited to infinitesimal values of $t_\perp$, while our results show that *large* values of $t_\perp$ are still unable to produce a difference between bonding and antibonding states if

$\alpha > 1$. In fact, we believe that the results of Fig. 3 are the first direct numerical evidence of a strong effect of interactions on hopping between chains because they exhibit a qualitative change without having to go to the thermodynamic limit. Besides, our results concerning the dependence of ΔE on N show very dramatic effects for relatively small interactions. These conclusions agree qualitatively with those of Refs. [1] and [6], and they are consistent with previous numerical studies of the spectral functions [9,7].

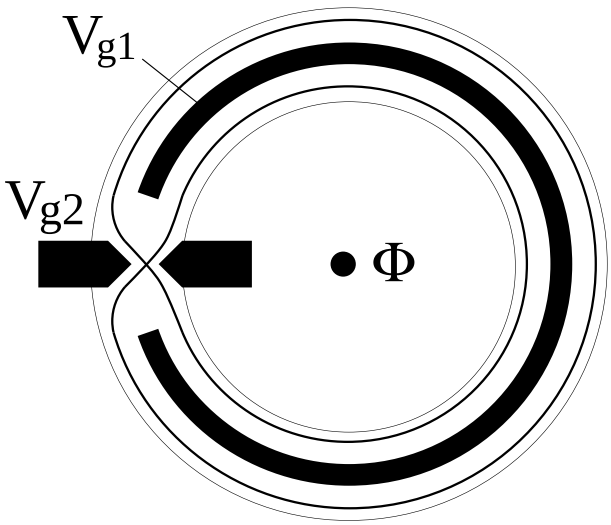

FIG. 5. Possible realization of a Moebius ring using a wide quantum wire. At low densities, electrons will form Luttinger liquids along the edges of the wire. Interedge tunneling is controlled by gate g1, while the crossover between the two edges is controlled by gate g2.

TThe main advantage of the present approach is that the quantities we calculate are not spectral functions, whose low-energy behaviour is very difficult to measure with today's resolution, but thermodynamic quantities that should be accessible in experiments on mesoscopic systems. A Moebius ring with controllable interedge tunneling could in principle be fabricated in a GaAs heterostructure: At sufficiently low carrier densities, electrons in a wide quantum wire will form Luttinger liquids along the two edges, and interedge tunneling could be controlled via an external gate g1 (see Fig. 5). The crossover between the two edges could be provided by a ballistic cross, controlled by gate g2, as illustrated in Fig. 5. The persistent current in such a structure, which is related to the ground state energy by $I(\Phi) = -(e/\hbar)\partial E/\partial\Phi$, could be determined experimentally via magnetization measurements [10]. $I(\Phi)$ can be expressed as a Fourier series, $I(\Phi) = \sum_{n=1}^{\infty} I_n \sin(n\Phi)$, and such experiments typically measure the first two harmonics, I_1 and I_2. The first harmonic is in fact proportional to ΔE,

$$I_1 = (-1)^{\frac{n-1}{2}} \frac{e\Delta E}{2\hbar N} + \mathcal{O}(I_3), \tag{5}$$

provided the higher odd harmonics can be neglected (which is always the case experimentally). ¿From the above discussion of ΔE, it is clear that I_1 should be a highly sensitive function of $t_\perp$ and of the carrier density in the coherent tunneling regime, while $I_1 = 0$ in the absence of coherent interedge tunneling. Experiments on suitably fabricated mesoscopic systems, in which the predicted finite-size effects could be directly measured, are thus good candidates to test theories of the effect of interactions on interchain tunneling.

In conclusion, we have shown that interactions between electrons have dramatic effects on their ability to hop coherently from one chain to the other by studying the flux dependence of the ground-state energy – or equivalently of the persistent currents – in a Moebius ladder. The absence of oscillations as a function of $t_\perp$ for strong enough interactions is, we believe, the best evidence of an interaction-induced destruction of interchain coherence obtained so far with numerical simulations. Further work along these lines, either numerically by studying larger systems, or experimentally by measuring persistent currents in appropriate mesoscopic devices, should be a promising area for future research.

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